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Stability of a Schwarzschild Singularity

TULLIO REGGE, *Istituto di Fisica della Università di Torino, Torino, Italy*

AND

JOHN A. WHEELER, *Palmer Physical Laboratory, Princeton University, Princeton, New Jersey*

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It is shown that a Schwarzschild singularity, spherically symmetrical and endowed with mass, will undergo small vibrations about the spherical form and will therefore remain stable if subjected to a small nonspherical perturbation.

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EFFECTIVE POTENTIAL FOR EVEN-PARITY REGGE-WHEELER GRAVITATIONAL PERTURBATION EQUATIONS*

Frank J. Zerilli

Physics Department, University of North Carolina, Chapel Hill, North Carolina 27514

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The Schrödinger-type equation for odd-parity perturbations on a background geometry has been extended to the even-parity perturbations. This should greatly simplify the analysis for calculations of gravitational radiation from stars and from objects falling into black holes.

Perturbations in vacuum - general setup

Consider $R_{\mu\nu}[g] = 0$

Let $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$, with being $\bar{g}_{\mu\nu}$ an exact solution

Now in Einstein equations $\delta R_{\mu\nu}[\delta g] = 0$ expand $\delta g_{\mu\nu} = \sum_i \varepsilon^i h^{(i)}_{\mu\nu}$ itself and get the hierarchy of perturbative Einstein equations (expression for $\delta R_{\mu\nu}$ contains all powers of $\delta g_{\mu\nu}$):

$$\Delta_L h^{(i)}_{\mu\nu} = S^{(i)}_{\mu\nu}$$

Thus, we trade **nonlinearities** of Einstein equations for an **infinite system of linear inhomogeneous equations** (the sources $S^{(i)}_{\mu\nu}$ are constructed from metric perturbations $h^{(j)}_{\mu\nu}$, with $j < i$). To solve it one needs:

1. a general solution of a principal (homogeneous) part
2. a particular solution of inhomogeneous part

$$\Delta_L {}^{(i)}h_{\ell\, tt} = \left[\frac{(2A+rA')^2 - 2(rA')^2 + 2(\ell-1)(\ell+2)A}{4r^2 A} + \left(\frac{A'}{4} - \frac{A}{r} \right) \partial_r - \frac{A}{2} \partial_{rr} \right] {}^{(i)}f_{\ell\, tt} + \left[\frac{AA'}{2} A \partial_r - \partial_{tt} \right] {}^{(i)}f_{\ell\, +}$$

$$- A \left[\frac{(2A+rA')^2 - 4A}{4r^2} + \frac{AA'}{4} \partial_r + \frac{1}{2} \partial_{tt} \right] {}^{(i)}f_{\ell\, rr} + \left[\left(\frac{A'}{2} + \frac{2A}{r} \right) \partial_t + A \partial_{tr} \right] {}^{(i)}f_{\ell\, tr} \,,$$

$$\Delta_L {}^{(i)}h_{\ell\, rr} = \left[\frac{4A(1-A) + (rA')^2}{4r^2 A^3} - \frac{A'}{4A^2} \partial_r + \frac{1}{2A} \partial_{rr} \right] {}^{(i)}f_{\ell\, tt} - \left[\left(\frac{A'}{2A} + \frac{2}{r} \right) \partial_r + \partial_{rr} \right] {}^{(i)}f_{\ell\, +}$$

$$+ \left[\frac{(2A+rA')^2 + 2A\left(2rA' + (\ell-1)(\ell+2)\right)}{4r^2 A} + \left(\frac{A'}{4} + \frac{A}{r} \right) \partial_r + \frac{1}{2A} \partial_{tt} \right] {}^{(i)}f_{\ell\, rr} - \left(\frac{A'}{2A^2} \partial_t + \frac{1}{A} \partial_{tr} \right) {}^{(i)}f_{\ell\, tr} \,,$$

$$\Delta_L {}^{(i)}h_{\ell\, tr} = \frac{A}{r} \partial_t {}^{(i)}f_{\ell\, 11} + \left[\left(\frac{A'}{2A} - \frac{1}{r} \right) \partial_t - \partial_{tr} \right] {}^{(i)}f_{\ell\, +} + \frac{\ell(\ell+1)}{2r^2} {}^{(i)}f_{\ell\, tr} \,,$$

$$\Delta_L {}^{(i)}h_{\ell\, +} = \left[-\frac{2rA' + \ell(\ell+1)}{4A} + \frac{r}{2} \partial_r \right] {}^{(i)}f_{\ell\, tt} + \frac{A}{2} \left[\left(2A + 3rA' + \frac{\ell(\ell+1)}{2} \right) + rA\partial_r \right] {}^{(i)}f_{\ell\, rr}$$

$$+ \frac{1}{2} \left[(\ell-1)(\ell+2) - r(4A + rA')\partial_r - r^2 A \partial_{rr} + \frac{r^2}{A} \partial_{tt} \right] {}^{(i)}f_{\ell\, +} - r\partial_t {}^{(i)}f_{\ell\, tr} \,,$$

$$\Delta_L {}^{(i)}h_{\ell\, t\theta} = \frac{1}{2} \left[\left(A' + A\partial_r \right) {}^{(i)}f_{\ell\, tr} - A\partial_t {}^{(i)}f_{\ell\, rr} - \partial_t {}^{(i)}f_{\ell\, +} \right] ,$$

$$\Delta_L {}^{(i)}h_{\ell\, r\theta} = \frac{2A + rA'}{4r} {}^{(i)}f_{\ell\, rr} - \frac{1}{2} \partial_r {}^{(i)}f_{\ell\, +} - \frac{1}{2A} \partial_t {}^{(i)}f_{\ell\, tr} + \frac{1}{2A} \left(-\frac{2A + rA'}{2rA} + \partial_r \right) {}^{(i)}f_{\ell\, tt} \,,$$

$$\Delta_L {}^{(i)}h_{\ell\, -} = \frac{1}{4} \left(\frac{1}{A} {}^{(i)}f_{\ell\, tt} - A {}^{(i)}f_{\ell\, rr} \right) .$$